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A FUZZY LINEAR PROGRAMING APPROACH IN PRODUCTION PLANNING

BY

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Abstract. Nowadays fast and right decision making has become more and more important for both people and enterprises. Several optimization problems are, in fact, linear problems. Few or most of the involved parameters or correlations of them are imprecise or uncertain due to the time pressure on the companies' decision makers or due to the limited capabilities of information processing. Decision – making process in a fuzzy environment represents, in fact, a kind of decision process where the goals and/or the constraints are fuzzy themselves. This paper targets to solve out fuzzy linear programming problem by Zimmerman approach. To highlights its applicability, this paper considers a numerical example in order to determine what the monthly production should be in order maximize the profit. WinQSB Softwere is used.

Keywords: linear programming; fuzzy sets theory; fuzzy goals; fuzzy constraints; tolerance limits.

1. Introduction

Linear programming (LP) is the most frequently applied operations research technique. A LP model represents real world situations with some sets

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of parameters determined by experts and decision makers while in real world applications certainty, reliability and precision are often illusory concepts, therefore experts and decision makers cannot determine the exact value of parameters, or it could be the case that they cannot specify the objective functions or constraints precisely.

The classical LP problem is stated as:

$$\begin{aligned} &\text{Max} && c^T x \\ &\text{such that,} && Ax \leq (\geq) b, \\ & && x \geq 0 \end{aligned} \tag{1}$$

in which $x \in \mathbb{R}^n$, $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$ where A, b and c are crisp numbers andthe symbols ≤ and ≥ have the precise sense and "max" is certain maximum meaning.

In most of the cases it is not practical to define the constraints and the objective function in crisp terms and hence application of fuzzy linear programming (FLP) offers the advantage that the decision maker can model the problem in accordance to the current state of information. When decision is to be made in a fuzzy environment, there may be one of the three following modifications to LP.

First, the objective function is not to be maximized. In other word some aspiration level is to achieve which is not crisply definable as optimal. For example, the actual purpose may be to improve the profit situation.

Second, the constraints might be vague. The ≤ or ≥ signs might not be defined in the traditional sense and not be meant in the strict mathematical sense but to the degree that some violation may be acceptable. This can happen when the constraints represent aspiration levels that are not definable crisply. The decision maker may accept some violation of different constraints.

Finally, data might be inexact because of lack of precision or some vagueness about the data taking part procedure. Thus, the matrix A and the vectors $b$and $c$may not be crisp but rather may be fuzzy numbersand inequality may be interpreted as a ranking of fuzzy numbers.

In the literature, (Shams *et al*., 2012), according to the mentioned possible modification of LP model, FLP has been classified into three different categories:

1) FLPs with fuzzy constraints and crisp objective function
2) FLPs with fuzzy constraints and objective function
3) FLPs with fuzzy constraints and fuzzy coefficients

In this paper, we solving the FLP problems with fuzzy constraints and objective function.

2. FLPs with Fuzzy Constraints and Objective Function

The general model of an LP problem with fuzzy constraints and objective function is as follows:

$$\begin{array}{ll} \widetilde{\operatorname{Max}}_{x \in \mathbb{R}_+^n} & c^T x \\ \text{such that,} & Ax \lesssim (\gtrsim) b, \\ & x \geq 0 \end{array} \tag{2}$$

where the symbols "$\lesssim$ and $\gtrsim$" denote the fuzzified versions of "$\leqslant$ and $\geqslant$" and can be read as "approximately less/greater than or equal to and $\widetilde{\text{Max}}$ represents fuzzy maximizing and has the linguistic interpretation "satisfy aspiration level z_0 as best as possible". The symbol A denotes the $m \times n$ matrix A. In a production scheduling problem, c would indicate the n costs, $A_{m \times n}$ the matrix of technical coefficients, the m resources, and x the n variables. We call this model "type 2 FLP".

To solve such a problem, we review the method which is proposed by Zimmermann (Zimmermann, 2001).

The algorithm consists of three main steps:

1) Define the membership functions (MFs) and determine the fuzzy feasible set,

2) Determine the fuzzy set of the optimal value,

3) Solve using the maxi-min operator.

3. Zimmermann's Approach

Zimmermann used linear MFs and min operator as an aggregator for these functions, and assigned an equivalent LP problem to FLPs with fuzzy constraints and objective function.

In this approach, z_0 is determined by decision maker.

Find x such that

$$\begin{array}{c} c^T x \gtrapprox z_0 \\ A_i x \lesssim b_i, (i = \overline{1, m}) \\ x \geq 0 \end{array} \tag{3}$$

where A_i represents line i of the A matrix. In some problems restrictions as below could come up:

$$A_i x \gtrsim b_i \tag{4}$$

To solve (3), we should first choose an appropriate MF for each of the fuzzy inequality and then employ Bellman and Zadeh principle (Bellman and Zadeh, 1970) to identify the fuzzy decision.

At step 1, let the i^{th} resource b_i, $i = \overline{1, m}$ being defined by the interval $[b_i, b_i + p_i]$ with tolerance p_i, where p_i is also determine by decision maker. If $A_i x \leq b_i$, the i^{th} constraint is satisfied. If $A_i x \geq b_i + p_i$, the i^{th} constraint is violated. For $A_i x \in (b_i, b_i + p_i)$ the MF is monotonically decreasing.

We consider, for restrictions type (4), b_i, $i = \overline{1, m}$ being defined by the interval $[b_i - q_i, b_i]$ with tolerance q_i, where q_i is also determine by decision maker. If $A_i x \geq b_i$, the constraint is satisfied. If $A_i x \leq b_i - q_i$, the constraint is violated. For $A_i x \in (b_i - q_i, b_i)$ the MF is monotonically decreasing.

Based on these ideas, according to the resolution methods proposed in (Zimmermann, 2001), (3) will become

$$\begin{gathered} c^T x \gtrapprox z_0 \\ A_i x \lessapprox b_i + p_i, (i = \overline{1,m}) \\ x \geq 0 \end{gathered} \tag{5}$$

If in (3) there are also restrictions of the type (4) then at (5) the following line will be added:

$$A_i x \gtrsim b_i - q_i \tag{6}$$

Let $\mu_i (i = \overline{1,m})$ denote the MF for the first i^{th} constraint. Then Zimmermann proposed nondecreasing and continuous linear MFs as follows:

$$\mu_i(A_i x) = \begin{cases} 1 & , A_i x < b_i \\ 1 - \frac{A_i x - b_i}{p_i} & , b_i \leq A_i x \leq b_i + p_i \\ 0 & , A_i x > b_i + p_i \end{cases} \tag{7}$$

For restrictions of the type (6) the continuous linear MFs are as follows:

$$\mu_i(A_i x) = \begin{cases} 0 & , A_i x < b_i - q_i \\ 1 - \frac{b_i - A_i x}{q_i} & , b_i - q_i \leq A_i x \leq b_i \\ 1 & , A_i x > b_i \end{cases} \tag{8}$$

At step 2, let μ_0 denote the MF for the objective function. Decision maker, also determine p_0 be the admissible tolerances for the objective function. Zimmermann proposed nondecreasing and continuous linear MF as follows:

$$\mu_0(c^T x) = \begin{cases} 1 & , c^T x > z_0 \\ 1 - \frac{z_0 - c^T x}{p_0} & , z_0 - p_0 \leq c^T x \leq z_0 \\ 0 & , c^T x < z_0 - p_0 \end{cases} \tag{9}$$

At step 3, using the “min” operator (Gasimov and Yenilmez, 2002) together with the above membership functions, μ_0 as objective membership function and $\mu_i (i = \overline{1,m})$as the membership function for the all i^{th} constraint, the “type 2 FLP” problem converted to find x^* in decision space, so we have:

$$\mu_D(x) = \max_{x \geq 0} \min_{i = \overline{0,m}} \mu_i(x) \tag{10}$$

By introducing one new variable, λ, which corresponds essentially to “type 2 FLP”, this leads to the following crisp LP problem:

$$\begin{aligned} \max_{x,\lambda} \quad & \lambda \\ \textit{such that,} \quad & c^T x \geq z_0 - (1-\lambda) p_0 \\ & A_i x \leq b_i + (1-\lambda) p_i, \\ & A_i x \geq b_i - (1-\lambda) q_i, \\ & \lambda \in [0,1], x \geq 0 \end{aligned} \tag{11}$$

Let's consider (x^*, λ^*)as an optimal solution of the transformed (crisp) linear programming problem. In such situationx^* is considered to be an optimal solution of the problem "type 2 FLP" and λ^* is the degree up to which the aspiration level z_0 of the decision maker is met.

4. Numerical Example

Let's consider a hypothetical numerical example (Rațiu-Suciu *et al.*, 2002) in order to show the method described before. The aim of this application is to determine monthly production planning and profit of a company firstly by LP model with certain data and secondly to determine interactively by Zimmermann's approach and finally to have a comparison among these models.

In a company, the general director intends to manufacture three new products A, B and C by using two raw materials RM1and RM2. 1800 units of RM1 and 1600 units of RM2 are available for production. The requirement of raw materials by each product is given below:

Table 1
The Requirement of Raw Materials by Each Product

Raw material	Requirement per unit of product		
	A	B	C
RM1	3	1	3
RM2	1	2	4

It is knowing that maximum 400 units from product A, 400 units from product B and 300 units from product C could be sold. The profit for a unit of products A, B and C sale are about 12, about 10 and about 16monetary units (hereinafter referred as m.u.), respectively. The director needs to determine how many products A, B and C should be manufactured in order to maximize the total profit.

The scope is to determine monthly production and profit of factory. Let's consider x_1 as the quantity of product A that will be produced, x_2 as the quantity of product B and x_3 is the quantity of product C. Then, the mathematical model of the production is:

$$\begin{gathered}
\text{Maximize } 12x_1 + 10x_2 + 16x_3 \\
\text{such that} \\
3x_1 + x_2 + 3x_3 \leq 1800 \\
x_1 + 2x_2 + 4x_3 \leq 1600 \\
x_1 \leq 400 \\
x_2 \leq 400 \\
x_3 \leq 300 \\
x_i \geq 0, i = \overline{1,3}
\end{gathered} \tag{12}$$

To solve this problem linear programming will be run by a computer product WinQSB 2.0 (Chang and Desai, 2003). The Linear Programming modules leads to the optimal value of the objective function. It is determined as 10044.44 m.u. for $x_1 = 355.55, x_2 = 400$ and $x_3 = 111.11$. Resources actually used are: $(1800, 1600)$. The findings are presented in Table 2.

Table 2

The Results of LP Obtained with the Computer Product WinQSB

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	355.5555	12.0000	4,266.6670	0	basic	4.0000	16.0000
2	X2	400.0000	10.0000	4,000.0000	0	basic	6.2222	M
3	X3	111.1111	16.0000	1,777.7780	0	basic	12.0000	22.8000
	Objective	Function	(Max.) =	10,044.4400				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1,800.0000	<=	1,800.0000	0	3.5556	1,000.0000	1,900.0000
2	C2	1,600.0000	<=	1,600.0000	0	1.3333	1,466.6670	2,166.6670
3	C3	355.5555	<=	400.0000	44.4444	0	355.5555	M
4	C4	400.0000	<=	400.0000	0	3.7778	60.0000	600.0000
5	C5	111.1111	<=	300.0000	188.8889	0	111.1111	M

Taking into account the solution previously found, the director further asks for the optimal mix of production in order to get a profit around 12000 m.u., but not less than 11200 m.u. It is considered that during the manufacturing process the resources might be enhanced by not more than 600 units of each product type. Moreover, the director asks that the not fulfilled demand never to exceed 100 pieces of each product type (A, B or C, respectively).

Using Zimmermann's approach, the mathematical model of the problem is:

$$\begin{gathered} \text{Maximize } \lambda \\ \text{such that} \\ 12x_1 + 10x_2 + 16x_3 - 800\lambda \geq 11200 \\ 3x_1 + x_2 + 3x_3 + 600\lambda \leq 2400 \\ x_1 + 2x_2 + 4x_3 + 600\lambda \leq 2200 \\ x_1 - 100\lambda \geq 300 \\ x_2 - 100\lambda \geq 300 \\ x_3 - 100\lambda \geq 200 \\ \lambda \in [0,1] \\ x_i \geq 0, i = \overline{1,3} \end{gathered} \tag{13}$$

To solve the Linear programming problem a *WinQSB computer product*, *Linear Programming module* is required. This gives the optimal value of $x_1 = 361.90, x_2 = 336.50, x_3 = 236.50$ and $\lambda = 0.3651$. The solution is presented in Table 3.

Table 3
The Results of FLP Obtained with the Computer Product WinQSB

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	361.9048	0	0	0	basic	-0.0017	0.0008
2	X2	336.5079	0	0	0	basic	-0.0100	0.0012
3	X3	236.5079	0	0	0	basic	-0.0100	0.0034
4	niu	0.3651	1.0000	0.3651	0	basic	0	M
	Objective	Function	(Max.) =	0.3651				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	11,200.0000	>=	11,200.0000	0	-0.0001	10,953.8500	11,438.4600
2	C2	2,350.7940	<=	2,400.0000	49.2063	0	2,350.7940	M
3	C3	2,200.0000	<=	2,200.0000	0	0.0010	1,816.6670	2,293.9400
4	C4	325.3968	>=	300.0000	25.3968	0	-M	325.3968
5	C5	300.0000	>=	300.0000	0	-0.0011	276.6917	345.7143
6	C6	200.0000	>=	200.0000	0	-0.0025	179.4702	236.3636
7	C7	0.3651	<=	1.0000	0.6349	0	0.3651	M

This production mix leads to an income of $(11200 + 0.365 \times 800) = 11492$ m.u. It requires $(2400 - 49.2063 - 0.365 \times 600) = 2131.7937$ units of RM1 resource and $(2200 - 0.365 \times 600) = 1981$ units of RM2 resource.

The original linearized objective function has the optimum value of 10044.44 m.u. The best decision occurs at $x_1 = 361.90, x_2 = 336.50, x_3 = 236.50$ and $\lambda = 0.3651$ resulting in an optimum value of 11492 u.m. representing an increase of about 14.41%. This optimal solution obtained from the FLP incorporates uncertainties in the objective function and constraints. The maximizing grade of membership is $\lambda = 0.3651$. This value can be considered to be a measure of the degree of acceptability of this optimal decision.

5. Conclusions

Manufacturing processes takes place within an environment featured by varying degrees of uncertainties: the degree of the data certainty, reliability and precision are usually low the production processes. Hence, the incorporation of vague and imprecise data into the operations research techniques help to the improvement of the solution in most production management problems.

In the first step of the solution process, the system is modelled by using only the information which the decision maker provides without any expensive acquisition. Knowing the first comprehensive solution, the decision maker can incorporate further information in the constraints and objective function to improve the optimality.

The physical format of LP is turned into a fuzzy based LP format by converting into fuzzy goal, duly introducing flexibility into the constraints. The objective function value has increased by about 14.41% for the problem presented due to the application of fuzzy approach compared to the conventional LP approach. The solutions obtained through this integrated approach are more realistic for deciding a proper course of decision making, although the computational effort is little more than of deterministic solutions.

The acceptability of 0.3651 indicates the degree to which each constraint is satisfied under the uncertain conditions, thus allows decision maker to identify the needed flexibility for various constraints.

This paper has discussed the use of FLP for solving a production planning problem. This problem was solved by Zimmerman approach. It is just one of the FLP's approaches since the model has fuzziness in both objective function and constraints. The solution determines the optimum quantity of each type of the product to be manufactured in order to get maximum profit out of it.

REFERENCES

Bellman R.E., Zadeh L.A., *Decision-Making in a Fuzzy Environment*, Management Science, **17**, *4*, 141-164 (1970).

Chang Y.-L., Desai K., WinQSB Version 2.0, John Wiley & Sons, 2003.

Gasimov R.N., Yenilmez K., *Solving Fuzzy Linear Programing Problems with Linear Membership Functions*, Turk Journal Math., 267, 375-396 (2002).

Rațiu-Suciu C., Luban F., Hîncu D, Ene N., *Modelare ecomomică aplicată. Studii de caz. Teste*, București, Editura Economică, 2002.

Shams H., Mogouee M.D., Jamali F., Haji A., *A Survey on Fuzzy Linear Programming*, Americam Journal of Scientific Research, 75, 117-133 (2012).

Zimmermann H.J., *Fuzzy Set Theory and Applications*, 4rd ed. Kluwer Academic Publisher, 2001.

O METODĂ DE PROGRAMARE LINEARĂ FUZZY PENTRU PLANIFICAREA PRODUCȚIEI

(Rezumat)

În prezent luarea rapidă și corectă a deciziilor a devenit din ce în ce mai importantă atât pentru oameni cât și pentru întreprinderi. Mai multe probleme de optimizare sunt, în fapt, probleme liniare. Mai mulți sau mai puțini parametri implicați sau corelațiile dintre aceștia sunt imprecise sau incerte datorită presiunii timpului exercitat asupra factorilor de decizie din firmelor sau datorită capacităților limitate de procesare a informațiilor. Procesul de luare a deciziilor într-un mediu fuzzy reprezintă, de fapt, un proces de decizie în care obiectivele și/sau constrângerile sunt ele însele fuzzy. Această lucrare are ca obiectiv rezolvarea problemei de programare liniară fuzzy prin abordarea Zimmerman. Pentru a evidenția aplicabilitatea sa, lucrarea prezintă un exemplu numeric determinând producția lunară optimă în vederea maximizării profitului. Este utilizat softul WinQSB.